Face Load Factor $K_{H\beta}$ for Planetary Stages

In the issue 2006 of ISO6336-1, the equations for calculating the face load coefficients for planetary gear units have been omitted. The standard contains the statement that “there are some commonly arising conditions which are not covered by the equations of 7.5.2.4 but which can be easily dealt with by slight variants on those equations”. However, the modifications needed are certainly not "slight." Above all, creating variants of this kind can result in discussions. The number of large planetary gear units being produced has risen considerably in recent years, so it is already regrettable that formulae for this specific case are unobtainable.

The principal equations to obtain the face load factor are (eq. 41 and 52 in ISO6336-1):

$$K_{H\beta} = \frac{(Fm/b)_{max}}{Fm/b} = 1 + \frac{F_{\beta_s} \cdot c_\beta}{2 \cdot Fm/b}$$  \hspace{1cm} (1)

$$F_{\beta_s} = F_{\beta_p} \cdot \kappa_\beta = (1.33 \cdot f_{sh} + f_{mu}) \cdot \kappa_\beta$$  \hspace{1cm} (2)

The following description shows the derivation of $f_{sh}$ for planets, strictly based on the philosophy of ISO 6336-1. Appendix D of the standard describes the derivation of the formulae which are applied for a single gear pair.

The deformation component $f_{sh}$ is derived from the deformation of the mating gears on the shaft due to torsion and bending. In order to simplify the situation for a pinion-gear pair, only the pinion deformation (which is much greater) is taken into account. Appendix D of the standard describes how the deformation is calculated. Because only the change in the deformation across the face width is of interest, the deformation component on the left-hand end (origin) is set to zero.

Planetary stages are subject to the following major deformations:

- Since the sun has several tooth meshings, all radial forces are canceled out. No bending takes place, so the deformation is caused solely by torsion. However, the multiple meshing, which corresponds to the number of planets, means this is much greater than for normal pinion shafts.
- A planet has two meshings with opposed torques, which prevents deformation due to torsion. Bending may be calculated in the same way as for pinion shafts. However, the circumferential force must be doubled because of the sun/planet and planet/internal gear meshing.
- In most cases, rim deformation can be ignored.

So, the torsion of the pinion and the bending of the planet shaft must be taken into consideration for sun/planet meshing whereas, for planet/internal gear meshing, only the bending of the planet shaft is important.
For most support arrangements for planets, bending can be determined analytically using a procedure similar to that specified in ISO 6336-1. Figure 1 shows the four most common cases.

Figure 1: Planet support types:
- a) Planets mounted with fixed clamped bolts on both sides
- b) Planets are on bolts, which have flexible bearings on planet carrier
- c) Planets mounted with gently tightened bolts (flexible bearings) on both sides
- d) Planets mounted with fixed clamped bolts on one side

Equations 3a to 3d show the bending component dependent on the distance \( x \) from the beginning of the face width on the planet. As only the change in the bending across the facewidth is of interest, the constant term has been omitted from the equations, so \( f(x = 0) \) is zero. Similar formulae can be found in the technical literature [2]. For cases a through d according to figure 1, the following applies:

\[
\begin{align*}
 f_{bpl\text{a}} &= 2 \frac{64 \ Fm/b}{\pi \ E_p d_{sh}^4} \ *[x^4 \ / \ 24 - x^3 b / 12 - x^2 b (3l - 6b + b^2 / l) / 48 + x b^2 (3l - 4b + b^2 / l) / 48] \quad (3a) \\
 f_{bpl\text{b}} &= 2 \frac{64 \ Fm/b}{\pi \ E_p d_{pl}^4} \ *[x^4 \ / \ 24 - x^3 b / 12 - x^2 b (l - b) / 8 + x b^2 (l / 8 - b / 12)] \quad (3b) \\
 f_{bpl\text{c}} &= 2 \frac{64 \ Fm/b}{\pi \ E_p d_{sh}^4} \ *[x^4 \ / \ 24 - x^3 b / 12 - x^2 b (l - b) / 8 + x b^2 (l / 8 - b / 12)] \quad (3c) \\
 f_{bpl\text{d}} &= 2 \frac{64 \ Fm/b}{\pi \ E_p d_{sh}^4} \ *[x^4 \ / \ 24 - x^3 b / 6 + x^2 b^2 / 4 + x b (l - b) / 2] \quad (3d)
\end{align*}
\]
The sun's deformation due to torsion, according to equation 4, can be calculated from Appendix D (fi as specified in formula D.1).

\[ f_{tso} = p \frac{8}{\pi} \frac{Fm/b}{0.39E_{so}} \cdot (\frac{b}{d_{so}})^2 \cdot \frac{x}{b} \cdot (1 - \frac{x}{2b}) \]  

(4)

<table>
<thead>
<tr>
<th>Formula symbol</th>
<th>Unit</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>( b )</td>
<td>mm</td>
<td>Meshing face width</td>
</tr>
<tr>
<td>( c_{y\beta} )</td>
<td>N/(mm ( \mu )m)</td>
<td>Meshing stiffness</td>
</tr>
<tr>
<td>( d_{pla} )</td>
<td>mm</td>
<td>Planet reference circle</td>
</tr>
<tr>
<td>( d_{sh} )</td>
<td>mm</td>
<td>Planet shaft diameter</td>
</tr>
<tr>
<td>( d_{so} )</td>
<td>mm</td>
<td>Sun reference circle</td>
</tr>
<tr>
<td>( E_p )</td>
<td>N/mm(^2)</td>
<td>Young's modulus planet bolt/shaft</td>
</tr>
<tr>
<td>( E_{so} )</td>
<td>N/mm(^2)</td>
<td>Young's modulus sun</td>
</tr>
<tr>
<td>( f_{pla} )</td>
<td>mm</td>
<td>Planet shaft deflection</td>
</tr>
<tr>
<td>( f_{2\beta} )</td>
<td>( \mu )m</td>
<td>Helix slope deviation in accordance with ISO 1328</td>
</tr>
<tr>
<td>( f_{ma} )</td>
<td>( \mu )m</td>
<td>Tooth trace deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>manufacture error</td>
</tr>
<tr>
<td>( f_{sh} )</td>
<td>( \mu )m</td>
<td>(Linearized) deformation components of the tooth trace deviation</td>
</tr>
<tr>
<td>( f_{tco} )</td>
<td>mm</td>
<td>Sun torsion deviation</td>
</tr>
<tr>
<td>( F_{m/b} )</td>
<td>N/mm</td>
<td>Average line load</td>
</tr>
<tr>
<td>((F_{m/b})_{max})</td>
<td>N/mm</td>
<td>Maximum local line load</td>
</tr>
<tr>
<td>( F_{by} )</td>
<td>( \mu )m</td>
<td>Actual tooth trace deviation</td>
</tr>
<tr>
<td>( K_{2\beta} )</td>
<td>[-]</td>
<td>Face load factor</td>
</tr>
<tr>
<td>( l )</td>
<td>mm</td>
<td>Planet bolt/shaft length</td>
</tr>
<tr>
<td>( p )</td>
<td>mm</td>
<td>Number of planets</td>
</tr>
<tr>
<td>( x )</td>
<td>mm</td>
<td>Distance to the left side of the facewidth</td>
</tr>
<tr>
<td>( K_{gb} )</td>
<td>[-]</td>
<td>Run-in factor</td>
</tr>
</tbody>
</table>
To comply with the method used in ISO 6336-1 as closely as possible (and to enable formula 2 to be applied), the average deformation components \( f_{\text{bmpla}} \) (bending at the planet) and \( f_{\text{tmso}} \) (torsion at the sun) are determined.

\[
f_{\text{bmpla}} = \frac{1}{b} \int_0^b f_{\text{iso}}(x) \cdot dx \quad f_{\text{tmso}} = \frac{1}{b} \int_0^b f_{\text{pla}}(x) \cdot dx
\]

\[
f_{\text{bmpla}} = 2 \frac{64}{\pi} \frac{Fm/b}{E_p d_{sh}} \frac{b^3}{16} \left( -\frac{b}{5} + \frac{l}{6} + \frac{b^2}{18l} \right)
\]

\[
f_{\text{bmpla}} = 2 \frac{64}{\pi} \frac{Fm/b}{E_p d_{pla}} \frac{b^3}{16} \left( \frac{l}{3} - \frac{b}{5} \right)
\]

\[
f_{\text{bmpla}} = 2 \frac{64}{\pi} \frac{Fm/b}{E_p d_{sh}} \frac{b^3}{16} \left( \frac{l}{3} - \frac{b}{5} \right)
\]

\[
f_{\text{bmpla}} = 2 \frac{64}{\pi} \frac{Fm/b}{E_p d_{pla}} \frac{b^2}{4} \left( \frac{b^2}{5} - b \cdot l + l^2 \right)
\]

\[
f_{\text{iso}} = p \frac{8}{3\pi} \frac{Fm/b}{0.39E_{so}} \left( \frac{b}{d_{so}} \right)^2
\]

As specified in equation D.8 in the standard, the linearized deformation components of the tooth trace deviation \( f_{\text{sh}} \) (in \( \mu \text{m} \)) now looks like this:

\[
f_{\text{sh}} \text{(Paarung Sonne - Planet)} = 2000 \cdot (f_{\text{iso}} + f_{\text{bmpla}})
\]

\[
f_{\text{sh}} \text{(Paarung Planet - Ring)} = 2000 \cdot f_{\text{bmpla}}
\]

This can then be used with equations (2) and (1) to calculate face load factors for the sun/planet and planet/gear rim meshing.